

Math 115

Spring 2017

Lecture 26

① Simplify: $\frac{2x-5}{x^2-5x+6} - \frac{x-1}{x^2-5x+6} = \frac{2x-5-x+1}{x^2-5x+6}$

$$= \frac{x-4}{x^2-5x+6}$$

② Simplify: $\frac{x+1}{x^2+5x+6} - \frac{x+3}{x^2-4}$

$$= \frac{(x+1)(x-2)}{(x+3)(x+2)(x-2)} - \frac{(x+3)(x+3)}{(x+2)(x-2)(x+3)}$$

$$= \frac{\cancel{x^2} - 2x + x - 2 - \cancel{x^2} - 3x - 3x - 9}{(x+3)(x+2)(x-2)} = \frac{-7x-11}{(x+3)(x+2)(x-2)}$$

③ Find all the excluded values:

$$\frac{2x-7}{3x^2-2x-5}$$

$$\boxed{\text{E.V. : } -1 \text{ \& } 5/3}$$

$$3x^2-2x-5=0$$

$$(3x-5)(x+1)=0$$

$$3x-5=0$$

$$x+1=0$$

$$\boxed{x=5/3}$$

$$\boxed{x=-1}$$

④ Reduce : $\frac{x^3-3x^2-4x+12}{x^2-5x+6}$

$$= \frac{x^2(x-3)-4(x-3)}{(x-2)(x-3)} = \frac{(x-3)(x^2-4)}{(x-2)(x-3)} = \frac{\cancel{(x-3)}(x+2)\cancel{(x-3)}}{\cancel{(x-2)}\cancel{(x-3)}} = \boxed{x+2}$$

⑤ Divide:

$$\frac{x^2-7x+12}{2x^2+7x+5} \div \frac{x^2-16}{4x^2-25}$$

$$= \frac{(x-3)\cancel{(x-4)}}{\cancel{(2x+5)}(x+1)} \cdot \frac{\cancel{(2x+5)}(2x-5)}{(x+4)\cancel{(x-4)}}$$

$$= \boxed{\frac{(x-3)(2x-5)}{(x+1)(x+4)}}$$

Simplify: $\frac{x - \frac{9}{x}}{1 + \frac{3}{x}} = \frac{x \cdot x - x \cdot \frac{9}{x}}{x \cdot 1 + x \cdot \frac{3}{x}}$

$LCD = x = \frac{x^2 - 9}{x + 3} = \frac{(x+3)(x-3)}{x+3}$

Simplify:

$2 + \frac{3}{x} - \frac{5}{x^2} = \frac{2x^2 + 3x - 5}{x^2} = \frac{(2x+5)(x-1)}{x^2}$

$\frac{2}{x} + \frac{5}{x^2} = \frac{2x + 5}{x^2} = \frac{(2x+5)(x-1)}{x^2}$

$LCD = x^2 = \boxed{x-1}$

Simplify: $\frac{1}{x - x^{-1}}$

Hint: $x^{-n} = \frac{1}{x^n}$

$= \frac{1}{x - \frac{1}{x}} = \frac{x}{x^2 - 1}$

$LCD = x$

Simplify:

$\frac{5x^{-1} - 2y^{-1}}{25x^{-2} - 4y^{-2}} =$

$\frac{\frac{5}{x} - \frac{2}{y}}{\frac{25}{x^2} - \frac{4}{y^2}} = \frac{x^2 y^2 \cdot \frac{5}{x} - x^2 y^2 \cdot \frac{2}{y}}{x^2 y^2 \cdot \frac{25}{x^2} - x^2 y^2 \cdot \frac{4}{y^2}} = \frac{5xy^2 - 2x^2y}{25y^2 - 4x^2}$

$LCD = x^2 y^2 = \frac{xy(5y - 2x)}{(5y - 2x)(5y + 2x)} = \boxed{\frac{xy}{5y + 2x}}$

Solving Rational Equations:

- ① find LCD & E.V.
- ② Multiply by LCD to clear fractions
- ③ Solve the new equation.
- ④ only keep those solutions that are not excluded values.

Solve: $\frac{x}{10} - \frac{10}{x} = 0$ LCD = $10x$, E.V.: 0

~~$10x \cdot \frac{x}{10} - 10x \cdot \frac{10}{x} = 0$~~
 $x^2 - 100 = 0$

$(x + 10)(x - 10) = 0$
 by Z.F.T.
 $x = -10, x = 10$ $\{\pm 10\}$

Solve: $\frac{1}{x+2} + \frac{1}{x-2} = \frac{4}{x^2-4}$

LCD = $(x+2)(x-2)$

E.V.: $-2, 2$

~~$(x+2)(x-2) \cdot \frac{1}{x+2} + (x+2)(x-2) \cdot \frac{1}{x-2} = (x+2)(x-2) \cdot \frac{4}{x^2-4}$~~

~~$x-2 + x+2 = 4$~~

$2x = 4 \rightarrow \boxed{x=2}$



Solve: $\frac{3}{x-7} - \frac{1}{x+4} = \frac{2}{x^2-3x-28}$

LCD = $(x-7)(x+4)$, E.V.: 7 & -4

$$3(x+4) - 1(x-7) = 2$$

$$3x + 12 - x + 7 = 2$$

$$2x + 19 = 2$$

$$2x = -17$$

$$\boxed{x = -\frac{17}{2}} \checkmark$$

$$\left\{ -\frac{17}{2} \right\}$$

Solve $\frac{x+1}{x+3} + \frac{x-3}{x-2} = \frac{x^2-11x}{x^2+x-6}$

LCD = $(x+3)(x-2)$ E.V.: 2 & -3

$$(x-2)(x+1) + (x-3)(x+3) = x^2 - 11x$$

FOIL

FOIL

$$x^2 - x - 2 + \cancel{x^2} - 9 - \cancel{x^2} + 11x = 0$$

$$x^2 + 10x - 11 = 0$$

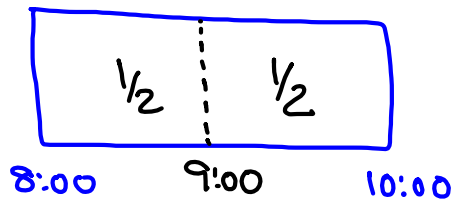
$$(x+11)(x-1) = 0$$

$$\boxed{x = -11}$$

$$\boxed{x = 1}$$

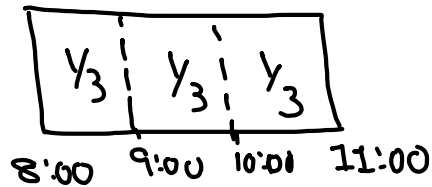
$$\left\{ -11, 1 \right\}$$

work Problems:



Paint this wall

2 hrs

Your rate $\frac{1}{2}$ /hr.

Paint this wall

3 hrs

My rate is $\frac{1}{3}$ /hr.

How long if we work together?

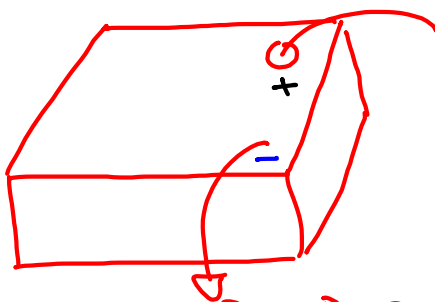
$$\frac{1}{2} \cdot t + \frac{1}{3} \cdot t = 1 \text{ Complete work}$$

$$3t + 2t = 6$$

$$\frac{t}{2} + \frac{t}{3} = 1$$

$$\text{LCD} = 6$$

$$t = \frac{6}{5} \quad \boxed{t = 1.2 \text{ hrs}}$$



Pipe A can fill it up in 4 hrs.

Rate $\rightarrow \frac{1}{4}$ /hr.Rate $\rightarrow \frac{1}{6}$ /hr.

Pipe B can empty it in 6 hrs.

both pipes working on a empty pool, How long to

fill up the pool? $\frac{1}{4} \cdot t - \frac{1}{6} \cdot t = 1$

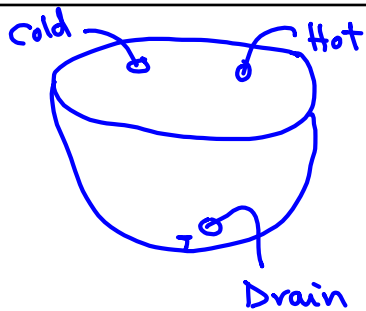
$$\text{Solve } \frac{t}{4} - \frac{t}{6} = 1$$

$$\text{LCD} = 12$$

$$3t - 2t = 12$$

$$\boxed{t = 12}$$

$$\boxed{12 \text{ hrs}}$$



Hot water in 8 mins.

Cold water in 5 mins.

Drain can empty the Sink in

$$t = \frac{40}{9} \approx 4.5 \text{ mins.}$$

Sink is empty, Drain is open, both water running, how long before the sink is full?

Solve

$$\underbrace{\frac{1}{8} \cdot t}_{\text{Hot}} + \underbrace{\frac{1}{5} \cdot t}_{\text{Cold}} - \underbrace{\frac{1}{10} \cdot t}_{\text{Drain}} = 1$$

$$\frac{t}{8} + \frac{t}{5} - \frac{t}{10} = 1$$

$$\text{LCD} = 40$$

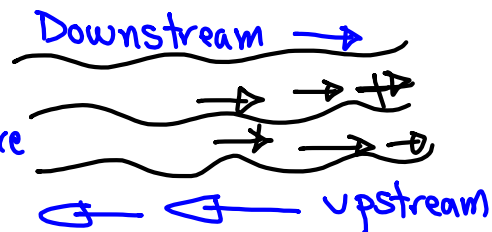
$$5t + 8t - 4t = 40$$

$$9t = 40$$

A boat can travel 6 miles upstream in the same time that it can travel 10 miles downstream. the current is 3 mph.

Find Speed for boat.

Dist.	Rate · time = distance
Upstream	$x - 3 \cdot t = 6$
Downstream	$x + 3 \cdot t = 10$



$$t_{\text{upstream}} = \frac{6}{x-3}$$

Same time $\frac{6}{x-3} = \frac{10}{x+3}$

$$10(x-3) = 6(x+3) \rightarrow x = 12$$

$$10x - 30 = 6x + 18$$

$$4x = 48$$

$$\boxed{12 \text{ mph}}$$

$$t_{\text{Downstream}} = \frac{10}{x+3}$$

A small plane flying @ 16 mph.

It went 48 miles with wind in the Same ^{time} that it took to go 16 miles into wind. find the rate of wind.

Cat.	r	t	= d
with wind	$16 + x$	t	$= 48$
into wind	$16 - x$	t	$= 16$

$t_{\text{wind}} = t_{\text{against wind}}$

$$\frac{48}{16+x} = \frac{16}{16-x}$$

$$\frac{3}{16+x} = \frac{1}{16-x}$$

$$3(16-x) = 1(16+x)$$

$x = 8$

8 mph

Alicia drove 180 miles in the same time that Jon drove 120 miles. Alicia was driving 20 mph faster than Jon. Find speed for both.

Cat.	r	t	= d
Alicia	$x+20$	t	$= 180$
Jon	x	t	$= 120$

$t_{\text{Alicia}} = t_{\text{Jon}}$

$$\frac{180}{x+20} = \frac{120}{x}$$

Jon 40 mph

Alicia 60 mph

$$x = 40 \quad \frac{3}{x+20} = \frac{2}{x}$$

Mario drove 50 miles in the mountain and 180 miles on the highway.

Entire trip was 5 hrs.

His speed on the HWY was 10mph more than ^{Twice} his speed in the mountain.

Find speed on each part.

Cat.	r	t	d
Mountain	x	$t = 50$	
HWY	$2x+10$	$t = 180$	

$$t_{\text{Mountain}} + t_{\text{HWY}} = 5$$

$$\frac{50}{x} + \frac{180}{2x+10} = 5$$

$$\frac{50}{x} + \frac{90}{x+5} = 5$$

Solve $\frac{50}{x} + \frac{90}{x+5} = 5$

Divide by 5

$$\frac{10}{x} + \frac{18}{x+5} = 1$$

$x=25$ ~~$x=-2$~~

25 mph in the mountain &
60 mph on the HWY

SG 19 due
Thursday

LCD = $x(x+5)$, E.V. 0, -5

$$10(x+5) + 18x = x(x+5)$$

$$10x + 50 + 18x = x^2 + 5x$$

$$28x + 50 = x^2 + 5x$$

$$x^2 + 5x - 28x - 50 = 0$$

$$x^2 - 23x - 50 = 0$$

$$(x-25)(x+2) = 0$$